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LETTER TO THE EDITOR

Chiral feedback for p-wave superconductors

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Abstract. In a quasi-two-dimensional p-wave superconductor we find six Cooper pairing states which are degenerate within the weak-coupling approach. We show that this degeneracy can be lifted by feedback effect favouring the so-called chiral p-wave state. This effect is based on the anomalous coupling between charge and current in a system with broken time reversal symmetry and parity.

The discovery of odd-parity Cooper pairing in Sr_2RuO_4 led to the renewed interest in the so-called p-wave (spin-triplet) superconductivity [1, 2]. There is a number of other systems, such as the heavy Fermion superconductors UPt_3 and UBe_{13} or the organic superconductor $(TMTSF)_2PF_6$, where odd-parity pairing is very likely realized [3, 4]. The spin-1 degree of freedom of the spin-triplet Cooper pairs provides a considerably wider space of potential pairing states than in the even-parity (spin singlet) case [5]. The symmetry and effective dimensionality of the electronic band structure plays an important role in determining the possible p-wave pairing states. The examples listed above cover many of the possibilities: the organic superconductor is quasi-one-dimensional, the heavy Fermion compounds are three-dimensional, while Sr_2RuO_4 represents the case of a quasi-two-dimensional system.

The large number of possible p-wave states makes their identification for each material a difficult task. A simple weak-coupling BCS type of approach can give a first guess of the most stable state. Because the condensation energy is in this case directly connected with the presence of the energy gap in the quasi-particle spectrum, the state with the least nodes in the gap would be most favourable. In one and three dimensions the most stable state is unique up to spin rotation. Assuming parabolic band structure for the corresponding dimension and using the $\boldsymbol{d}$-vector notation we find

$$\begin{aligned} \boldsymbol{d}(\boldsymbol{k}) &= \hat{\boldsymbol{x}} k_x && \text{for 1d} \\ \boldsymbol{d}(\boldsymbol{k}) &= \hat{\boldsymbol{x}} k_x + \hat{\boldsymbol{y}} k_y + \hat{\boldsymbol{z}} k_z && \text{for 3d} \end{aligned} \tag{1}$$

where the gap matrix is defined as $\hat{\Delta}_{\boldsymbol{k}} = i\sigma_2 \boldsymbol{\sigma} \cdot \boldsymbol{d}(\boldsymbol{k})$ and the quasiparticle gap is $\frac{1}{2}\mathrm{tr}[\hat{\Delta}^+_{\boldsymbol{k}} \hat{\Delta}_{\boldsymbol{k}}] = |\boldsymbol{d}(\boldsymbol{k})|^2$. Obviously the two states are nodeless on the corresponding Fermi surfaces. Note that the example for three dimensions corresponds to the Balian–Werthammer state or the B-phase of superfluid ^{3}He [6].

We now consider the case of as quasi-two-dimensional system which is characterized by the fact that the Fermi surface is open in one direction, the z-axis. In such a system the weak coupling approach does not lead to a unique state, but we can find six degenerate states with the same nodeless gap. In Sr_2RuO_4 their degeneracy is lifted by spin–orbit coupling and the corresponding states labelled according to the representation of the tetragonal crystal point group of this compound is given in table 1 [7, 8]. We can distinguish two types of states here:

those which have $\boldsymbol{d}$-vectors that change orientation for different points on the Fermi surface belonging to the one-dimensional representation A_{1u}, A_{2u}, B_{1u} and B_{2u} and those which have a fixed $\boldsymbol{d}$-vector orientation but a finite orbital angular momentum, belonging to the two-dimensional E_u representation [7, 8]. Note that the latter is the chiral state, i.e. it breaks time reversal symmetry and parity. Since all these states are degenerate in the spin rotation symmetric case beyond simple spin rotation transformation, the question arises which among them is stable. For Sr_2RuO_4 experiments suggest the chiral state with $\boldsymbol{d} \parallel \hat{\boldsymbol{z}}$ [2, 9].

Table 1. Six-fold degenerated states in p-wave pairing symmetry.

Γ	$\boldsymbol{d}(\boldsymbol{k})$
A_{1u}	$\hat{\boldsymbol{x}}\hat{k}_x + \hat{\boldsymbol{y}}\hat{k}_y$
A_{2u}	$\hat{\boldsymbol{x}}\hat{k}_y - \hat{\boldsymbol{y}}\hat{k}_x$
B_{1u}	$\hat{\boldsymbol{x}}\hat{k}_x - \hat{\boldsymbol{y}}\hat{k}_y$
B_{1u}	$\hat{\boldsymbol{x}}\hat{k}_y + \hat{\boldsymbol{y}}\hat{k}_x$
E_u (chiral states)	$\hat{\boldsymbol{z}}(\hat{k}_x \pm i\hat{k}_y)$

In Sr_2RuO_4 the loss of spin rotation symmetry by spin–orbit coupling carries the main responsibility in picking the stable state. In this letter, however, we assume that the spin rotation symmetry is preserved in the normal state so that all states listed in table 1 have the same transition temperature T_c as a solution of the linearized weak-coupling gap equation. In this case the degeneracy must be lifted in a higher order process. A well-known concept introduced by Anderson and Brinkman is the spin fluctuation feedback mechanism [10, 11]. If paramagnon exchange plays a dominant role in the pairing interaction, the modification of the spin fluctuation spectrum by the superconducting condensation also alters the pairing interaction. It was shown that this mechanism works in favour of the so-called AMB-state or A-phase in ^{3}He [6]. This mechanism applied to the 2D situation turns out to stabilize the time reversal breaking state which is indeed the analogue to the A-phase [12, 13].

Here we introduce an additional feedback mechanism which does not exist in a neutral Fermi liquid such as ^{3}He. It is based on the presence of chirality in the orbital part of the pairing state and we will call it, therefore, the *chiral feedback mechanism*. It was shown that in the state $\boldsymbol{d}(\boldsymbol{k}) = \boldsymbol{n}(k_x \pm ik_y)$ a Chern–Simons-like term,

$$\epsilon_{ij}(A_0\partial_i A_j + A_i\partial_j A_0) \tag{2}$$

appears in the effective low-energy field theory of a static quasi-two-dimensional system ($i, j = x, y$). [14, 15] Consequently, charge fluctuations generate local magnetic field distributions (z-axis oriented) and current fluctuations lead to transverse electric field distributions, whose orientation depends on the chirality. This property yields an additional (anomalous) pairing interaction in the superconducting state which has selective power for chirality. This can be seen in the following simple picture. The magnetic field induced by the charge of a quasiparticle acts via the Lorentz force on a passing-by quasiparticle [16]. This force is either attractive or repulsive depending on which side the quasiparticle trajectory is located (figure 1). In this way the interaction is attractive for one choice of Cooper pair angular momentum (chirality) and repulsive for the other. The attractive interaction appears for the same chirality realized in the pairs of the condensate. Hence this leads to a positive feedback for the chiral state. This effect does not exist for the other states.

We now discuss the effect by explicitly calculating its contribution to condensation energy immediately below the transition temperature T_c. We represent the $\boldsymbol{d}$-vector by $\boldsymbol{d}(\boldsymbol{p}) = \boldsymbol{n}_\gamma d_{\gamma i} k_i / k_{\mathrm{F}}$ where $d_{\gamma i}$ is a complex order parameter and k_{F} inverse of the Fermi wavelength. The band structure is simply parabolic $\epsilon_{\boldsymbol{k}} = \hbar^2(k_x^2 + k_y^2 - k_{\mathrm{F}}^2)/2m_e$ without any

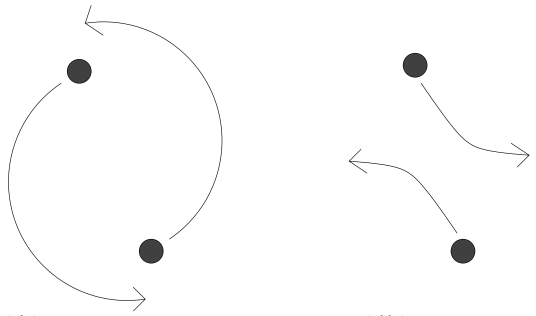

Figure 1. Two quasiparticles feel (i) attractive or (ii) repulsive interaction depending on their relative angular momentum.

dispersion along the z-axis leading to a cylindrical (open) Fermi surface. We assume that the system has a layered structure as Sr_2RuO_4 with an interlayer spacing d leading to the density of states, $N(0) = m_e/2\pi\hbar^2 d$, at the Fermi level. The anomalous pairing interaction appearing in the superconducting phase is connected with the density-current correlation function which for the 2D electrons has the form (in the unit $\hbar = c = 1$)

$$\pi_{0j}(i\nu_n, \boldsymbol{q}) = \int \mathrm{d}\tau \mathrm{d}^3 x \mathrm{e}^{\mathrm{i}\nu_n\tau} \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}} \langle \hat{\rho}(\boldsymbol{x}, \tau)\hat{j}_j(0,0)\rangle$$
$$= k_B T \sum_m \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{-(2k_j + q_j)}{2m_e} \mathrm{Tr}[\mathcal{G}(i\omega_m + i\nu_n, \boldsymbol{k}+\boldsymbol{q})\mathcal{G}(i\omega_m, \boldsymbol{k})$$
$$-\mathcal{F}^\dagger(i\omega_m + i\nu_n, \boldsymbol{k}+\boldsymbol{q})\mathcal{F}(i\omega_m, \boldsymbol{k})] \quad (j = 1, 2) \tag{3}$$

and the Green functions are

$$\mathcal{G}(i\omega_m, \boldsymbol{k}) = \frac{i\omega_m + \epsilon(\boldsymbol{k})}{\omega_m^2 + E(\boldsymbol{k})^2} \qquad \text{and} \qquad \mathcal{F}(i\omega_m, \boldsymbol{k}) = -\frac{i\Delta(\boldsymbol{k})}{\omega_m^2 + E(\boldsymbol{k})^2} \tag{4}$$

with $E(\boldsymbol{p}) = \pm\sqrt{\epsilon(\boldsymbol{k})^2 + \Delta(\boldsymbol{k})^2}$ and $\omega_m = (2m+1)\pi k_B T$ and $\nu_n = 2n\pi k_B T$ as the fermionic and bosonic Matsubara frequencies, respectively. We express $\pi_{0j}(i\nu_n, \boldsymbol{q}) = i\epsilon_{ij} q_i f(i\nu_n, \boldsymbol{q}) + \nu_n q_j \pi(i\nu_n, \boldsymbol{q})$ due to the gauge invariance. $f(i\nu_n, \boldsymbol{q})$ comes from the chirality and is written as

$$f(i\nu_n, \boldsymbol{q}) = -\frac{i}{2!}\epsilon_{ij} \frac{\partial \pi_{0j}(i\nu_n, \boldsymbol{q})}{\partial q_i}. \tag{5}$$

Close to T_c we can restrict ourselves to the leading contributions in $d_{\gamma i}$ and obtain,

$$f(i\nu_n, \boldsymbol{q}) = \frac{e^2 k_B T_c}{2m_e k_F^2} \sum_m \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{-i\epsilon_{ij} d_{\gamma l} d^*_{\gamma l'} k_l k_{l'}}{\{(\omega_m + \nu_n)^2 + \epsilon(\boldsymbol{k}+\boldsymbol{q})^2\}\{\omega_m^2 + \epsilon(\boldsymbol{k})^2\}} \tag{6}$$

where we sum over repeated indices. Note that this expression is only finite for a chiral p-wave state. We restrict ourselves to static contributions and comment on the dynamical part later.

We now consider the q-dependence. We approximate $\epsilon(\boldsymbol{k}+\boldsymbol{q}) = \epsilon(\boldsymbol{k}) + \boldsymbol{v}_\mathrm{F} \cdot \boldsymbol{q}$ where $\boldsymbol{v}_F$ is the Fermi velocity. In order to evaluate the integral we introduce cylindrical coordinates and perform first the integration over the radial part and z-component of k,

$$f(\boldsymbol{q}) = \frac{ie^2 k_B T_c}{4m_e} N(0)\pi(\epsilon_{ij} d_{\gamma i} d^*_{\gamma l}) \sum_m \int \frac{\mathrm{d}\theta}{2\pi} \frac{\hat{k}_l \hat{k}_j}{|\omega_m|(|\omega_m|^2 + \frac{1}{4}(\boldsymbol{v}_\mathrm{F}\cdot\boldsymbol{q})^2)} \tag{7}$$

where $\hat{k}_i = k_i/|\boldsymbol{k}|$. For small q we may expand $f(\boldsymbol{q})$ as

$$f(\boldsymbol{q}) \approx \frac{ie^2N(0)\pi}{4m_e(\pi k_B T_c)^2}\epsilon_{ij}d_{\gamma i}d^*_{\gamma j}\left[\frac{7}{4}\zeta(3) - \frac{31}{32}\zeta(5)\xi^2 q_\perp^2 + \cdots\right] \tag{8}$$

where $q_\perp^2 = q_x^2 + q_y^2$, $\xi = v_F/2\pi k_B T_c$ defines the coherence length and $\zeta(n)$ is the zeta-function. The behaviour of $f(\boldsymbol{q})$ for $\xi q_\perp \gg 1$ is dominated by the regions of the θ-integral for which $|\boldsymbol{v}_F \cdot \boldsymbol{q}| \ll 2\pi k_B T_c$ and leads to

$$f(\boldsymbol{q}) \approx \frac{ie^2}{4m_e}\frac{N(0)}{(\pi k_B T_c)^2}\frac{\epsilon_{ij}d_{\gamma i}d^*_{\gamma j}}{\xi q_\perp}. \tag{9}$$

Matching the limiting behaviours together we can approximate $f(\boldsymbol{q})$ by the following form,

$$f(\boldsymbol{q}) \approx \frac{ie^2}{4m_e}\frac{N(0)\pi}{(\pi k_B T_c)^2}\frac{\epsilon_{ij}d_{\gamma i}d^*_{\gamma j}}{\sqrt{1+\gamma\xi^2 q_\perp^2}} \qquad \gamma = \mathcal{O}(1) \tag{10}$$

which represents the form factor of parity and time reversal symmetry breaking part in $\pi_{0j}(0, \boldsymbol{q})$.

The current-charge density interaction introduced via $\pi_{0j}(0, \boldsymbol{q})$ gives an additional contribution to the pairing interaction below the superconducting transition. As a feedback effect this appears in the GL free energy in a fourth-order correction expressed by

$$\Delta F_{\mathrm{fb}} = k_B T_c \int \frac{\mathrm{d}^3 q}{(2\pi)^3} D_{00}(\boldsymbol{q})\pi_{0i}(\boldsymbol{q})D_{ij}(\boldsymbol{q})\pi_{j0}(\boldsymbol{q}), \tag{11}$$

following the diagram in figure 2. Here D_{00}, D_{ij} $(i, j = 1, 2)$ is the gauge field propagators which in Coulomb gauge are,

$$D_{00}(\boldsymbol{q}) = \frac{1}{q^2 + l_{\mathrm{TF}}^{-2}} \qquad \text{and} \qquad D_{ij}(\boldsymbol{q}) = \frac{-\delta_{ij}}{q^2} \tag{12}$$

These propagators contain all renormalizations, i.e. Thomas–Fermi screening for the scalar potential with the screening length l_{TF}. Since $T \approx T_c$ London screening of the superconductor can be neglected. Here we also ignore the dynamical part for simplicity, as it would give the same contributions for all competing states.

If we separate the q-integration in q_z- and $q_\perp$-part, we obtain,

$$\Delta F_{\mathrm{fb}} = \left\{\frac{e^2N(0)\pi}{4m_e(\pi k_B T_c)^2}\epsilon_{ij}d_{\gamma i}d^*_{\gamma j}\right\}^2 k_B T_c l_{\mathrm{TF}}^2 \times \int\frac{\mathrm{d}^2 q_\perp}{(2\pi)^2}\int\frac{\mathrm{d}q_z}{2\pi}\frac{q_\perp^2}{1+\gamma\xi^2 q_\perp^2}\frac{1}{q_\perp^2+q_z^2}\frac{1}{1+l_{\mathrm{TF}}^2(q_\perp^2+q_z^2)}. \tag{13}$$

After performing the q_z-integration the remaining $q_\perp$-integral has a cutoff $q_\perp \sim l_{\mathrm{TF}}^{-1}$. This leads to the free energy correction,

$$\Delta F_{\mathrm{fb}} \approx \frac{8\alpha^2}{\pi}\frac{T_c}{T_{\mathrm{F}}}\frac{l_{\mathrm{TF}}}{d}\frac{N(0)}{(\pi k_B T_c)^2}(\epsilon_{ij}d_{\gamma i}d^*_{\gamma j})^2 \tag{14}$$

where we give an expression formally close to the conventional fourth-order terms in order to give a comparison of its magnitude. Here we recover the constants $\hbar$ and c, the factor $\alpha = e^2/\hbar c$ is the fine structure constant and the ratio T_c/T_{F} indicates the strong coupling nature of the correction term, similar to the spin fluctuation feedback mechanism. [11]

For the chiral p-wave state $\epsilon_{ij}d_{\gamma i}d^*_{\gamma j} = i2\chi|\Delta|$ and zero for all other states ($\chi = \pm 1$ denotes the chirality). Thus, the correction to the fourth-order term is negative definite and

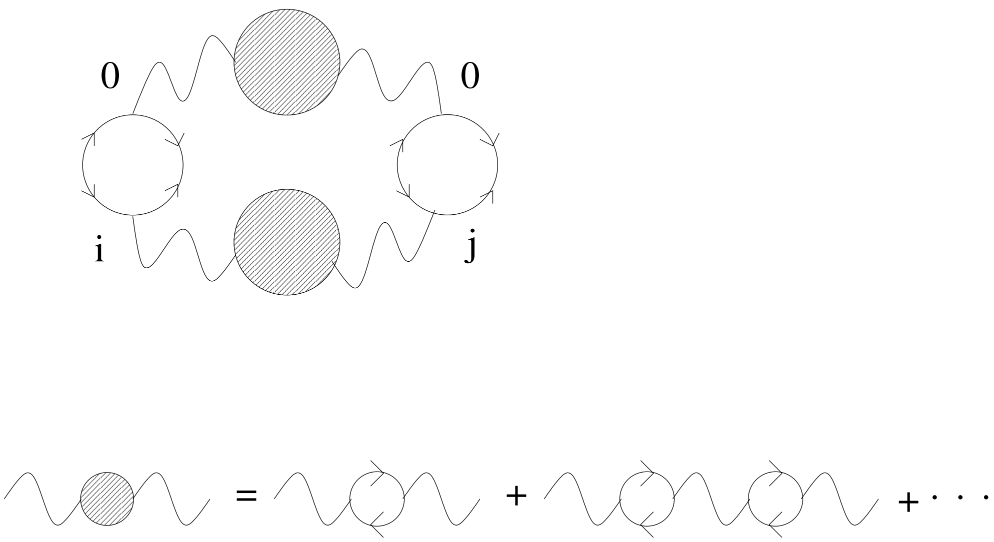

Figure 2. The diagram for the fourth order correction in GL free energy. The shadowed circles show the renormalization by the normal fermionic Green function.

favours the chiral p-wave state. The ratio between this correction and the usual fourth-order coefficient is

$$\frac{\delta\beta_{\mathrm{fb}}}{\beta} \sim \frac{\alpha^2}{\pi}\frac{T_c}{T_{\mathrm{F}}}\frac{l_{\mathrm{TF}}}{d} \tag{15}$$

which for Sr_2RuO_4 is of the order 10^{-6}.

It is easy to see that the dynamical contributions, taking into account $\nu_n \neq 0$, does not change the result qualitatively. The corresponding coefficient in the free energy, however, increases. We have verified numerically that an increase of one order of magnitude is possible. It is clear that other mechanisms, such as the spin fluctuation feedback or spin–orbit coupling, would dominate over the chiral feedback effect in stabilizing the chiral p-wave state. We would like to emphasize, however, that our analysis shows that for a quasi-two-dimensional p-wave superconductor the chiral feedback effect, based on the anomalous coupling between charge and current, supports the chiral superconducting phase and, thus, works in the same direction as the spin fluctuation feedback mechanism.

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